Math 106H **Final Exam** **Name _______________________.**

1. (12 points) Evaluate each of the following limits (no calculator):

   (a) $\lim_{x\to 2} \frac{x^2-7x+10}{x^2+2x-8}$

   (b) $\lim_{x\to\infty} \frac{3x^2-7x+10}{10x+x^2-100}$

   (c) $\lim_{x\to 0} \frac{1-\cos x}{2x^2}$.

2. (10 points) Solve each of the following:

   (a) $xy' = y, \quad y(2) = 4.$

   (b) $y' = \frac{\sin^2 x \cos^3 x}{y}.$

3. (11 points) Find the maximum and minimum values of $f(x) = x^3 - 12x$ on the closed interval $[-3, 5]$.

4. (12 points) (a) Use a tangent line approximation to approximate $(24.8)^{\frac{3}{2}}$.

(b) Use Newton's method to find the approximation x_2 for $\sqrt{8.8}$.

5. (20 points) Find the derivative of each of the following functions:

(a) $f(x) = x\sqrt{x} + 6^{x^2}$

(b) $f(x) = \sqrt[3]{x^2 + 1}$

(c) $h(y) = \ln \frac{y^2+1}{y^2-1}$

(d) $g(z) = \frac{3z+2}{2z-3}$

(e) $k(t) = (t^3 + 4)\sin^{-1}(t^2)$.

6. (12 points) Find an equation of the tangent line to the curve

$$x^3 + y^2 + x^2 y = y + 5,$$

at the point (1,2).

7. (12 points) Use the definition of the derivative to find $f'(3)$ given that

$$f(x) = \frac{1}{\sqrt{3x}}.$$

No credit for any other method.

8. (12 points) A region is bounded below by the parabola $y = x^2$ and bounded above by the line $y = x + 2$. Find a definite integral (don't evaluate) that gives the volume of the solid of revolution obtained by revolving this region about the line $y = 4$.

9. (16 points) Evaluate the following integrals:

   (a) $\int x^2 \cos(x^3) dx$

   (b) $\int \frac{x}{(1+x^2)^6} dx$

   (c) $\int \frac{x}{x^4+1} dx$

   (d) $\int (x+2)\sqrt{x+4} dx$.

10. (11 points) Draw a possible graph of a function $f(x)$ that satisfies the following: $f'(-1) = f'(2) = 0$, $f'(x) > 0$ on $(-\infty, -1)$ and $(-1, 2)$, $f'(x) < 0$ on $(2, \infty)$, $f''(x) > 0$ on $(-1, 0)$, and $f''(x) < 0$ on $(-\infty, -1)$ and $(0, \infty)$. (Clearly label your x-axis.)

11. (12 points) A circular cylindrical metal container, open at the top, is to have capacity of 24π cubic inches. The cost of the material used for the bottom of the container is 15 cents per square inch, and that of the material used for the curved part is 5 cents per square inch. If there is no waste of material find the dimensions that will minimize the cost of the material.

12. (12 points) A 20 foot rope that weighs 8 pounds hangs down 20 feet into a 30 foot hole in the ground. How much work do you do in pulling the rope to the top of the hole?

13. (12 points) Assume $f(x)$ is continuous on $[a, b]$. Define the definite integral $\int_a^b f(x)dx$. Also define the indefinite integral $\int f(x)dx$.

14. (12 points) Find a definite integral that gives the surface area of the surface of revolution obtained by rotating the curve $y = f(x) = e^{2x}$, $0 \leq x \leq 2$ about the x-axis. Use your calculator to evaluate the integral you obtain to five decimal places.

15. (12 points) Use Euler's method to approximate the value of the solution $y(x)$ of the IVP

$$y' = 2x + y, \quad y(1) = 2$$

at $x = 1.05$ and $x = 1.1$.

16. (12 points) Assume when a murder is committed the temperature of the body is 37 degrees C. Assume that after two hours the temperature of the body is 35 degrees C and assume that the temperature of the surrounding medium is always 20 degrees C. If the temperature of the body when it is found at 4:00 PM is 30 degrees C, when was the murder committed? Use Newton's law of cooling that says that the rate of change of the temperature of the body is proportional to the difference in the temperature of the body and the temperature of the surrounding medium.