

1. (18 points) Evaluate the following integrals:

a. $\int x^3 \ln x \, dx$

b. $\int \frac{3x^2 - x + 18}{x(x^2 + 9)} \, dx$

c. $\int \frac{1}{\sqrt{x^2 + 9}} \, dx.$

2. (16 points) Evaluate each of the following:

a. $\int_0^{\infty} x e^{-3x} \, dx$

b. $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x.$

3. (10 points) Write $.018181818 \dots$ as a fraction.

4. (35 points) Determine if each of the following infinite series converge or diverge (give reasons for your answers):

a $\sum_{k=1}^{\infty} \frac{k+2}{100k+15}$

b $\sum_{k=1}^{\infty} \frac{k^2+2}{k^3+1}$

c $\sum_{k=1}^{\infty} \frac{\sin k}{k\sqrt{k}}$

d $\sum_{k=1}^{\infty} \frac{k^2}{(2k+1)!}$

e $\sum_{k=1}^{\infty} (-1)^k \frac{1}{(2k+1)}$.

5. (15 points) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}3^k} x^k$.