

Math 108H Exam 1

Name _____.

Show all work, no calculator

1. (18 points) Find the derivative of each of the following:

a. $f(x) = x^2 - 2x\sqrt{x} - \frac{3}{x^3}$

b. $f(x) = \frac{x^2+3x-4}{x^2+9}$

c. $f(x) = e^{2x} \sec(x^3)$

2. (14 points)

a Find $\lim_{x \rightarrow \infty} \frac{2x^2+5x+13}{6+x+3x^2}$

b Find $\lim_{x \rightarrow 0} \frac{x^2}{1-\cos(3x)}$

3. (11 points) Use the definition of the derivative to find $f'(x)$ given that $f(x) = 2x^4 + x^2 - 100$.

4. (11 points) Use implicit differentiation to find all points on the curve $y^3 - xy = -6$ where the tangent line to the curve is either horizontal or vertical.

5. (18 points)

(a) Using an appropriate linear approximation, approximate $\sqrt{15.8}$.

(b) Use Newton's method one time to approximate $\sqrt[5]{33}$.

6. (12 points) A light is at the top of a 16-foot pole. A boy 5 feet tall walks away from the pole at a rate of 4 feet per second. At what rate is the tip of his shadow moving when he is 18 feet from the pole? At what rate is the length of his shadow increasing?

7. (16 points)

(a) Find $f'(x)$ if $f(x) = e^{\sqrt{x^2+10}}$.

(b) Find $f'(x)$ if $f(x) = \ln(\tan x)$.

(c) Find $f'(x)$ if $f(x) = \sin(e^{\cos(e^{4x})})$.

(d) State the Mean Value Theorem.