

1. (20 points) Evaluate each of the following iterated integrals:

a.  $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} \, dx dy$

b.  $\int_{-2}^0 \int_0^{\sqrt{4-x^2}} (x^2 + y^2)^{\frac{3}{2}} \, dy dx$

2. (11 points) Evaluate the triple integral  $\iiint_Q (x^2 + y^2 + z^2)^2 \, dV$ , where  $Q$  is bounded below by the surface  $z = \sqrt{3x^2 + 3y^2}$  and bounded above by top half of the sphere  $x^2 + y^2 + z^2 = 4$ .

3. (12 points) Find the extreme values of  $f(x, y) = 2x^2 + y^2$  on the circle  $x^2 + y^2 = 1$ .

4. (12 points) If  $f(x, y) = xe^y$ , find the rate of change of  $f$  at the point  $P = (2, 0)$  in the direction from  $P$  to  $Q = (\frac{1}{2}, 2)$ . In what direction does  $f$  have the maximum rate of change? What is this maximum rate of change?

5. (12 points) Find equations of the tangent plane and normal line at the point  $(-2, 1, -3)$  to the ellipsoid  $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$ .

6. (12 points) Find and classify the critical points for  $f(x, y) = x^4 + y^4 - 4xy + 1$ .

7. (12 points) Find the surface area of the part of the surface  $z = 2x + y^2$  that lies above the triangular region with vertices  $(0, 0)$ ,  $(1, 1)$ , and  $(0, 1)$ .

8. (11 points) Use cylindrical coordinates to evaluate the triple iterated integral

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{\frac{3}{2}} dz dy dx.$$