

Math 108H Final Exam
Show all work, no calculator

Name _____.

1. (16 points) Find the derivative of each of the following:

(a) $f(x) = x\sqrt{x} - \frac{3}{x^2}$

(b) $f(x) = \frac{x^2-4}{x^2+4}$

(c) $f(x) = x^2 \sin(3x)$

(d) $f(x) = \cos^5(4x)$

2. (11 points) Find the linearization of $f(x) = \sqrt{1+x}$ about $x = 0$.

3. (17 points) Evaluate the following limits:

(a) $\lim_{x \rightarrow \infty} \frac{x^2 + 4x - 7}{6 + x + 2x^2}$

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2}$

(c) $\lim_{x \rightarrow 2^+} \frac{5 - 3x}{x^2 - 4}$

(d) $\lim_{x \rightarrow \infty} \left(1 - \frac{4}{x}\right)^x$

4. (11 points) Use the definition of the derivative to find $f'(x)$ given that $f(x) = \frac{1}{x^2}$.

5. (11 points) Find an equation of the tangent line to the curve $x + y + 5 = 2xy + y^2$ at the point $(1, 2)$.

6. (11 points) Use Newton's method one time to approximate $\sqrt[3]{7.8}$.

7. (11 points) A rectangle has its base on the x -axis and its upper vertices on the curve $y = 12 - x^2$. What is the area of the largest such rectangle?

8. (11 points) Given $f(x) = 2x^4 - 4x^2 + 1$. Find intervals where f is increasing, is decreasing. Classify the critical points. Determine the concavity and find all points of inflection.

9. (11 points) The region bounded by the curves $y = 2x$ and $y = x^2$ is rotated about the line $x = 2$, find the volume swept out.

10. (27 points) Evaluate each of the following:

(a) $\int x e^{4x} dx$

(b) $\int x \sin(x^2 + 1) dx$

(c) $\int x^4 \ln x dx$

(d) $\int \frac{e^x}{\sqrt{e^x+4}} dx$

(e) $\int x \sqrt{x+1} dx$

(f) $\int \frac{1}{\sqrt{4x-x^2}} dx$

11. (11 points) Find the area of the region bounded by the curves $y = x^2 - 2x - 3$ and $y = x - 3$.

12. (11 points) Find the center of mass of a rod of length 3 meters with its left endpoint at $x = 0$ if its mass density is $\delta(x) = 1 + 2x$, $0 \leq x \leq 3$ kg per meter.

13. (11 points) Evaluate the integral $\int \frac{7x^2+x+17}{(x^2+4)(x-1)} dx$.

14. (11 points) Find the force on the bottom of the tank and on the inclined side of the tank pictured below if it is full of water.

15. (8 points) Evaluate $\int \frac{1}{(x^2+4)^2} dx$.

16. (11 points) A water tower is spherical in shape with radius 50 feet extended from 100 feet to 200 feet above the ground. How much work is done to fill the tank with water.