Math 107H Final Exam **Name** ______________________________.

1. (14 points) Given $f(x,y) = x^2 + 3x^2y^3 - y^3$.

   (a) Find the directional derivative of f at $(1,2)$ in the direction from $(1,2)$ towards the point $(2,1)$.

   (b) Find the minimum value of the directional derivatives of f at $(1,2)$.

   (c) Find a unit vector $\vec{v}$ so that the rate of change of f at $(1,2)$ in the direction of $\vec{v}$ is zero.

2. (12 points) Find an equation of the tangent plane and normal line to the surface $x^2 + 2y^2 = 27 - z^2$ at the point $(4, -1, 3)$.

3. (12 points) Find and classify the critical points of $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

4. (12 points) Find an equation of the plane passing through the point $P = (1, 0, 2)$ and perpendicular to the line of intersection of the planes $z = 2x - 3y + 4$, $z = x + y + 6$.

5. (15 points) Evaluate each of the following integrals:

(a) $\int x^3 \ln x \, dx$

(b) $\int x \sin(3x) \, dx$

(c) $\int \frac{3x^2-2x+5}{(x+1)(x^2+4)} \, dx$

6. (12 points) Evaluate the following:

(a) $\lim_{n\to\infty}(1 - \frac{2}{n})^{3n}$

(b) $\int_1^\infty \frac{x}{(8+x^2)^{\frac{3}{2}}} \, dx$

7. (12 points)

   (a) Find a parametric representation of the part of the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ in the upper half-plane that goes from $(0, 2)$ to $(-5, 0)$.

   (b) Given $w = f(x, y, z)$ and $x = g(r, s, t)$, $y = h(r, s, t)$, $z = k(r, s, t)$. Write down a chain rule formula for $\frac{\partial w}{\partial s}$.

8. (12 points) Determine if each of the following infinite series converge or diverge. Give reasons for your answers.

   (a) $\sum_{k=1}^{\infty} \frac{k+1}{3k+100}$

   (b) $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$

9. (12 points) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(-1)^k 2^k x^k}{k}$.

10. (11 points) Use Lagrange multipliers to find the point on the plane $2x + y - z = 5$ closest to the origin.

11. (12 points) Evaluate the following double iterated integral $\int_0^1 \int_y^1 x^2 e^{xy}\, dxdy$ by reversing the order of integration.

12. (12 points) Evaluate the double iterated integral $\int_{-1}^0 \int_0^{\sqrt{1-x^2}} 5\sqrt{x^2+y^2}\, dydx$.

13. (12 points) Using a triple integral, find the volume of the solid which is bounded above by the sphere $x^2 + y^2 + z^2 = 16$ and bounded below by the upper nappe of the cone $z^2 = 3(x^2 + y^2)$.

14. (14 points)

(a) Find the work done by the force field $\vec{F}(x, y) =< 2x + y, 1 + x >$ to move an object along the line segment from $(1, 2)$ to $(2, -3)$.

(b) Find the Taylor polynomial of degree three about $x = 0$ for $f(x) = \sqrt{1 - 2x}$.

15. (14 points) Given that $\vec{r} =< \cos t + t\sin t, \sin t - t\cos t >$ is the position vector of an object at time t. Find the velocity, speed, acceleration, normal component of acceleration, and tangential component of acceleration at time t. Find the radius of curvature of the curve at time t.

16. (12 points) Given $\vec{u} =< 2, 1, 3 >$, $\vec{v} =< -1, 2, 1 >$

    (a) Find the angle between $\vec{u}$ and $\vec{v}$.

    (b) Find the vector projection of $\vec{u}$ on $\vec{v}$.